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Mathematical Modelling of Marketing Dynamics: A Conceptualization of Ecological Growth Model in Fair and Unfair Competitions

NKUTURUM Christiana*
Department of Mathematics, Faculty of Science,
Rivers State University Nkpolu-Oroworukwo, Port Harcourt, Nigeria.

*Correspondence : NKUTURUM Christiana
christiana.nkuturum@ust.edu.ng

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Abstract : A nonlinear interaction model was proposed to study the dynamics of marketing by considering ecosystem in a trading environments that comprises two populations as fair competition – cheap goods and unfair –expensive products. The formulated model was analyzed to obtain the existence of equilibrium in bio-marketing ecosystem and dynamical behavior of equilibrium points which yielded a trivial and semi trivial solutions with conditions for stability and instability of the system. The system is unstable because the eigenvalue is greater than one. The system model revealed that there is fluctuation pattern as a result of the periodic functions and this was supported by the exact solutions and simulations. The study revealed that advertisement, sellers and buyers attitude which represents human action and diffusion significantly influenced the cheap goods more than the expensive products. In this study, table 2 describes the convergence point and the rate at which cheap goods enters the market in cartons of three, four and so on is higher than expensive products rate. The findings of this study supports decent work and economic growth, industry, innovation and infrastructure, responsible consumption and production, and partnerships for the goals that is the SDGs 8, 9, 12 and 17 respectively. MatLab ODE45 numerical scheme was used for the simulations.

Keywords: Cheap goods and expensive products, Ecological Growth Model, Fair and unfair competitions, Marketing Dynamics

Introduction

Marketing dynamics is the constant shifting forces including supply and demand fluctuations, consumer behaviour, competitive actions, and technology that influence market prices and business performance. It covers both real-time market changes and long-term, lagged effects of marketing actions, requiring strategic adaptation across the 4Ps - product, price, place and promotion [1-5]. Ecological growth models like exponential and logistic, are mathematical formulas used to predict population changes based on environmental resources that is geared towards creating an S-shaped curve that reflects, growth, resource limitation and carrying capacity thereby making it essential for conservation and resource management [11-13]. Fair competition thrives on merit, offering better products or prices honestly, while unfair competition uses deceptive, coercive or illegal tactics to gain an undue market advantage. The examples of unfair practices include misleading advertising, trademark infringement, trade secret theft, and disparagement, which violate ethical standards and consumer trust. Cheap goods often imply low initial monetary cost but may have hidden, long-term, or environmental expenses. In contrast, expensive products can range from high-quality, durable items to luxury, branded goods that command high prices despite similar functional performance to cheaper alternatives [6-9, 20-24].

Nowadays, scientists are looking at bridging, placing value and relationship between theory and practice in terms of mathematical modelling of real life problems. The problem of consumers behavior in a market economy has always been issues of concern to the right system thinking individuals on hoe to leverage business data analytics through mathematical modelling study to ascertain solutions to the dynamics sales forecast using time series [19,21].

Among other researchers, a mathematical modeling was developed to elucidate marketing dynamics, sales and promotion of small scale businesses taking over innovations for industrialization [10, 16]. A cord on an integrative decision-making mechanism for consumers' brand selection using 2-tuple fuzzy linguistic perceptions and decision heuristics by discretizing choice models for utility and probability in empirical Bayes estimation [17,18]. Brand metrics for Gauging and linking brands with business performance in marketing as a factor in consumer decision making and carried out network analysis of two-stage customer decisions with preference-guided market breakdown Munoz and Kumar developed an Online advertising response models incorporates multiple creative, Product life cycle assessment (LCA) as a tool for environmental management

Thus, Nugroho in the work [20] market segmentation analysis and positioning to increase product competitiveness in the global market, while [23, 24]. Recognized the relationships between fair and unfair competition among the pointers of marketing dynamics ecosystems in startups for national development while [6] critically reviewed the optimal pricing decision and capacity allocation of intelligence-based opaque selling in airline revenue management. Researchers are posed with some citations for better understanding [11-15, 25,]

Based on the above elucidated literature, this study proposed and developed a mathematical model on cheap and expensive goods in marketing ecosystems with a system of nonlinear differential equation. The study discussed the steady points, dynamical behavior of the points and obtained stability and instability of the system.

Methods
The model formulation

In this marketing dynamics of fair and unfair competitions model, the human action (selling, buying agents and advertisement) and Noise were identified in this study. However, we considered important components of the advertisement on the goods produced and sold in the market. These functional responses (advertisement) can be either fair competition-dependent or unfair competition-dependent. The functional response equation that is strictly unfair competition-dependent is the Holling group. Thus, Holling Tanner type of predator-prey model was considered where cheap goods C and expensive goods E denote the predator and prey. The parameter r_1 and r_2 represent increase in production rate, Q-quantity of expensive goods that enters the market per time and ρE-number of cheap goods that enters market per time, ρ-measure of the quality of expensive goods, $\delta E/(\beta + E)$ rate at which cheap goods outweighs expensive goods out of the market-the Holling-type II predator response, δ-maximum demand of expensive goods that can be sold per cheap goods per time, β-saturation value of expensive goods to achieve one half the maximum rate δ. Since the human action is positive on both competitions but more the cheap goods gives qhf-as the profit maximization, competition coefficient and force applied in marketing the goods and services. Based on the above elucidated marketing dynamics for fair-unfair competitions the formulation of mathematical model becomes

$$\frac{\partial C}{\partial t} = r_1 C\left(1 - \frac{C}{\rho E}\right) + \nabla^2 C + D_{12}\nabla^2 E \qquad 1$$

$$\frac{\partial E}{\partial t} = r_2 E\left(1 - \frac{E}{Q}\right) - \frac{\delta C E}{\beta + E} + \nabla^2 E + D_{21}\nabla^2 C \qquad 2$$

Where

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator in two–dimensional space. D_{12} and D_{21} are the diffusion coefficients. Here the two-dimensional space is the y and x coordinates depicting the dynamics (movement) of quantity of goods and services from the producers to the sellers and sellers to consumers vertically and horizontally respectively. In these competitions, the study considered the goods and services to be functions of several variables which is partial rate of change in cheap goods C with respect to time t and partial rate of change in expensive goods E with respect to time t. In this fair-unfair model, the forcing functions are: Human action, Noise and External Periodic Force. All the parameters are in non-dimensional form as to remove the units. Adding noise and periodic force terms to the model equation (1) and (2) becomes

$$\frac{\partial C}{\partial t} = r_1 C\left(1 - \frac{C}{\rho E}\right) + \nabla^2 C + D_{12}\nabla^2 E + qhfC - \mu C + A\sin(\omega t + \varphi) \qquad 3$$

$$\frac{\partial E}{\partial t} = r_2 E\left(1 - \frac{E}{Q}\right) - \frac{\delta CE}{\beta + E} + \nabla^2 E + D_{21}\nabla^2 C - \mu E + B\sin\omega t \qquad 4$$

The periodic force and noise is assumed to be sinusoidal with amplitude A, frequency ω and the phase shift φ in equation (3), while in equation (4), the periodic force and noise is also sinusoidal with amplitude denoted as B, frequency is ω. This periodic force is considered to be positive for the reason that pollutants produced by different sellers have a significant role in shaping the dynamical marketing behavior of ecosystems. In this study, we called the noise in equation (3) Green noise and Red noise is found in equation (4) are used to describe market place ecological progression. The noise in equation (3) will have an influence over that of equation (4).

$$\frac{\partial C}{\partial t} = r_1 C\left(1 - \frac{C}{\rho E}\right) + \nabla^2 C + D_{12}\nabla^2 E + qhfC - \mu C + A\sin(\omega t + \emptyset) \qquad 3$$

$$\frac{\partial E}{\partial t} = r_2 E\left(1 - \frac{E}{Q}\right) - \frac{\delta CE}{\beta + E} + \nabla^2 E + D_{21}\nabla^2 C - \mu E + B\sin\omega t \qquad 4$$

Model analysis

The nonlinear system of equations (1-2) are qualitatively analyzed as to ascertain the equilibrium points and the dynamical behavior and test for stability at positivity of equilibrium points.

Positivity analysis

Since (1) – (2) monitors density population of skill acquisition, all the state variables and parameters of the model are non- negative.

Lemma 1. If $C(t) \geq 0$, $E(t) \geq 0$ then the solution of C(t) and E(t) of the model equat (1) and (2) are positive.

Proof: From (1) $\frac{\partial C}{\partial t} = r_1 C\left(1 - \frac{C}{\rho E}\right) + D_{12}C\nabla^2 C + qhfC - \mu C + A\sin(\omega t + \varphi)$

Now, $\frac{\partial C}{\partial t} - r_1 C - qhf + U \geq 0 \Longrightarrow \frac{\partial C}{\partial t} + (\mu - r_1 - qhf)C \geq 0$

Let $P = \mu - r_1 - qhf$ and the integrating factor, IF$= \int e^{Pdt} = e^{Pt} = e^{(U - a_1 - Q\gamma\epsilon)t}$.

Multiply both sides by IF gives: $e^{(\mu - r_1 - qhf)t} e^{(\mu - r_{1-qhf})t} + e^{(\mu - r_{1-qhf})t}(\mu - r_1 - qhf)C \geq 0$

$\Longrightarrow e^{(\mu - r_1 - qhf)t} \geq A \Longrightarrow C(t) \geq Ae^{-(\mu - r_1 - qhf)t}$ (3)

Where A is an integer constant of integration. Applying the conditions when $t = 0$, $S(t) = S(0)$

$\therefore C(0) \geq A$. Substituting the value of A into (3) yields $C(t) \geq C(0)e^{-(U - a_1 - Q\gamma\epsilon)t}$. Since $C(t) \geq 0$ and $\mu - r_1 - qhf \geq 0$, it follows that $C(t) \geq 0$ for all $t \geq 0$. Similarly, from equation (2) $\frac{\partial E}{\partial t} = r_2 E\left(1 - \frac{E}{Q}\right) - \frac{\delta CE}{\beta + E} + D_{21}\nabla^2 E - \mu E + A_2\sin\omega t \quad \Longrightarrow \frac{\partial E}{\partial t} + \mu E - r_2 E \geq 0 \Longrightarrow \quad \frac{\partial E}{\partial t} + (\mu - r_2)E \geq 0$ and the integrating factor IF$= \int e^{(\mu - r_2)dt} = e^{(\mu - r_2)t}$.

Multiply both sides by IF gives: $e^{(\mu - a_2)t}\frac{\partial E}{\partial t} + e^{(\mu - a_2)t}(\mu - r_2)E \geq 0 \Longrightarrow \frac{\partial}{\partial t}\left(e^{(\mu - r_2)t}\right)E \geq 0$

$\Longrightarrow \left(e^{(\mu - a_2)t}\right)E \geq B \Longrightarrow E \geq Be^{-(\mu - a_2)t}$ (4)

Where B is an integer constant of integration. Applying the conditions when $t = 0$, $E(t) = E(0)$

$\therefore E(0) \geq B$. Substituting the value of B into (4) yields $E(t) \geq E(0)e^{-(\mu-a_2)t}$. Since $\mathrm{E}(t) \geq 0$ and $\mu - r_2 \geq 0$, it follows that $E(t) \geq 0$ for all $t \geq 0$. Therefore it is true that $\mathrm{C}(t)$ and $E(t)$ Are positive for all $t \geq 0$,

3.2. Existence of Equilibrium

The steady states of the system is obtained by setting $\frac{\partial C}{\partial t} = 0$ and $\frac{\partial E}{\partial t} = 0$
and solving algebraic equations gives two possible steady states as $\rho(0,0)$ and $\rho(C^*, E^*)$. The second solution is the positive solutions;

$$r_1 C\left(1 - \frac{C}{KE}\right) + D_{12}\nabla^2 e + qhfc - \mu C + A_1 \sin(\omega t + \varphi) = 0 \quad (5)$$
$$r_2 E\left(1 - \frac{E}{Q}\right) - \frac{\delta CE}{\beta + E} + D_{21}\nabla^2 C - \mu E + A_2 \sin \omega t = 0 \quad (6)$$

From (5), coefficient of cross diffusion is zero $D_{12} = 0, E = 0$ at equilibrium and the higher order terms of C^* is ignored and setting $E^* = 0$ so that

$$r_1 C^* + qhfC^* + A_1 \sin(\omega t + \varphi) = 0 \Longrightarrow (r_1 + Q\gamma\epsilon - \mu C)C^* = -A_1 \sin(\omega t + \varphi)$$
$$C^* = -\frac{A_1 \sin(\omega t + \varphi)}{(a_1 + Q\gamma\epsilon - US)} \quad (7)$$

Similarly, (6), coefficient of cross diffusion is zero $D_{21} = 0, S = 0$ at equilibrium and the higher order terms of H^* is ignored and setting $S^* = 0$ so that

$$(a_2 - \mu)E^* + A_2 \sin \omega t = 0 \Longrightarrow (a_2 - \mu)E^* = -A_2 \sin \omega t$$
$$E^* = -\frac{A_2 \sin \omega t}{(a_2 - \mu)} \quad (8)$$

From the above solutions, one of the equilibria solution is the trivial solution with non-acquisition of both skills; $\rho(0,0) = (0, 0)$ in a company where both skills are not yet existing. The other equilibrium solution revealed co-existence of both skills interacting for sustainable development of a nation as $\rho(C^*, E^*) = \left(-\frac{A_1 \sin(\omega t + \varphi)}{(a_1 + qhf - U)}, -\frac{A_2 \sin \omega t}{(a_2 - \mu)}\right)$. In the feasible equilibrium, it will interest you to understand that soft skills or hard skills acquisition population is independent of their growth or death rates in the company setting.

3.3 Dynamical behavior of equilibria

In this study, the dynamical behavior of equilibria can be calculated by computing vibrational matrices corresponding to each equilibrium. Note that the trivial equilibrium $\rho(0,0)$ is unstable. By applying the Routh-Hurwitz criteria all the eigenvalues of the vibrational matrix corresponding to ρ^* have negative real parts and ρ^* is locally asymptotically stable in the 2-D plane. This means that a small circle with centre ρ^* can be located such that any solution $(C(t), E(t))$ of system model equations which is inside the circle at some time $t = t_1$ will still remain inside the circle for all $t \geq t_1$ and will tend to (C^*, E^*) as $t \to \infty$.

Stability of steady states

Considering the fact that stability of deals with the equilibrium values, the study still maintains that cross diffusion is zero at equilibrium points. Suppose the two functions $u = u(C,E) = \frac{\partial C}{\partial t}$ and $v = v(C,E) = \frac{\partial E}{\partial t}$ from the steady state $(C^*, E^*) = (0,0)$ of the study model (1) and (2). Thus the Jacobian of the system gives-

$$J_{(\bar{s},\bar{h})} = \begin{Bmatrix} \frac{\partial u}{\partial C} & \frac{\partial u}{\partial E} \\ \frac{\partial v}{\partial C} & \frac{\partial v}{\partial E} \end{Bmatrix} = \begin{Bmatrix} a_1 - \frac{2a_1 s}{KE} + Q\gamma\epsilon - U & 0 \\ -\frac{\delta E}{\beta + E} & a_2 - \frac{2a_2 E}{P} - \frac{\delta S(\beta + E) - \delta CE}{(\beta + E)^2} - \mu \end{Bmatrix}$$

For the steady states $(\bar{C}, \bar{E}) = (0,0)$ gives,

(9)

$$J_{(0,0)} = \begin{Bmatrix} r_1 + qhf - \mu & 0 \\ 0 & r_2 - \mu \end{Bmatrix}$$

Hence the characteristics equation of the matrix with eigenvalue ω yields,

$$|J - \omega I| = \begin{vmatrix} a_1 + qhf - \mu - \omega & 0 \\ 0 & a_2 - \mu - \omega \end{vmatrix}$$

So that $r_1 + qhf - \mu - \omega = 0$ or $r_2 - \mu - \omega = 0$

$\therefore \omega_1 = r_1 + qhf - U, \omega_2 = r_2 - \mu$ (10)

However, the eigenvalue ω of the matrix determine the stability of the states. This study poses two conditions for stability depending on the value of ω: 1. If the eigenvalue $\omega < 0$, then the steady state is stable, 2. If the eigenvalue $\omega > 0$, then the system is unstable. $\omega_1 = r_1 + qhf - \mu > 0$, $\omega_2 = r_2 - \mu > 0$. Therefore the system is unstable because $\omega_1 > 0$ $and\ \omega_2 > 0$.

4 Exact solution

From (1) $\frac{\partial C}{\partial t} = r_1 c\left(1 - \frac{C}{KE}\right) + D_{12}\nabla^2 C + qhfC - \mu C + A_1 \sin(\omega t + \varphi)$

Now, $\frac{\partial C}{\partial t} - a_1 C - Q\gamma\epsilon C + UC = A_1 \sin(\omega t + \varphi) \Rightarrow$ $\frac{\partial C}{\partial t} - a_1 C - qhfC + \mu C \geq 0 \Rightarrow \frac{\partial C}{\partial t} + (\mu - a_1 - qhf)C \geq 0$

Let $P = \mu - r_1 - qhf$ and the integrating factor, IF$= \int e^{Pdt} = e^{Pt} = e^{(\mu - r_1 - qhf)t}$

$$\frac{d}{dt}(Ce^{pt}) = A_1 \sin(\omega t + \varphi)\,(e^{pt})$$

$$Ce^{Pt} = A_1 \int \sin(\omega t + \varphi) e^{Pt} dt$$

Put: $\int e^{Pt} \sin(\omega t + \varphi) dt = G \quad \Rightarrow Ce^{Pt} = A_1 G$

Integrating by parts $\int u dv = vu - \int v du$; Put $u = e^{Pt} \Rightarrow du = Pe^{pt}$

$dv = \sin(\omega t + \varphi) \Rightarrow v = -\frac{\cos(\omega t + \varphi)}{\omega}$

$$G = -\frac{e^{Pt}\cos(\omega t + \varphi)}{\omega} + \int \frac{P}{\omega}\cos(\omega t + \varphi) e^{Pt}\, dt$$

$$= -\frac{e^{Pt}\cos(\omega t + \varphi)}{\omega} + \frac{Pe^{Pt}}{\omega^2}\sin(\omega t + \varphi) + \frac{P}{\omega}\int \frac{P}{\omega}\sin(\omega t + \varphi) e^{Pt}\, dt$$

$$G = -\frac{e^{Pt}\cos(\omega t + \varphi)}{\omega} + \frac{Pe^{Pt}}{\omega^2}\sin(\omega t + \varphi) + \frac{GP^2}{\omega^2}$$

$$G\left(1 - \frac{P^2}{\omega^2}\right) = -\frac{e^{Pt}\cos(\omega t + \varphi)}{\omega} + \frac{Pe^{Pt}}{\omega^2}\sin(\omega t + \varphi)$$

$$G = -\frac{Pe^{Pt}\cos(\omega t + \varphi)}{\omega^2 + P^2} - \frac{Pe^{Pt}}{\omega^2 + P^2}\sin(\omega t + \varphi) + K$$

$$Ce^{Pt} = \frac{A_1 Pe^{Pt}\cos(\omega t + \varphi)}{\omega^2 + P^2} + \frac{Pe^{Pt}}{\omega^2 + P^2}\sin(\omega t + \varphi) + K$$

$C(t) = -\frac{A_1 P\cos(\omega t+\varphi)}{\omega^2+P^2} + \frac{P}{\omega^2+P^2}\sin(\omega t + \varphi) + Ke^{Pt}$

Initial condition $C(0) = K_1$ $(constant)$, from equation the above

$$-\frac{A_1 P\cos(\omega t + \varphi)}{\omega^2 + P^2} + \frac{P}{\omega^2 + P^2}\sin(\omega t + \varphi) + K = K_1$$

$$K = \frac{A_1 P\cos(\omega t + \varphi)}{\omega^2 + P^2} - \frac{P}{\omega^2 + P^2}\sin(\omega t + \varphi) + K_1$$

$C(t) = -\frac{A_1 P\cos(\omega t+\varphi)}{\omega^2+P^2} + \frac{P}{\omega^2+P^2}\sin(\omega t + \varphi) + e^{Pt}\left(\frac{A_1 \mathrm{P}\cos(\omega t+\varphi)}{\omega^2+P^2} - \frac{\mathrm{P}\sin(\omega t+\varphi)}{\omega^2+P^2} + K_1\right)$ (11)

Similarly, from euq (2)

$\frac{\partial E}{\partial t} - r_2 E + \mu E = A_2 \sin\omega t \Longrightarrow \quad \frac{\partial E}{\partial t} + \mu E - r_2 E \geq 0 \;\; ; \frac{\partial E}{\partial t} + (\mu - r_2)E \geq 0$

Let $Q = (\mu - a_2)$ and the integrating factor, IF= $\int e^{wdt} = e^{wt} = e^{(\mu - r_2)t}$

$\frac{d}{dt}(Ee^{wt}) = A_1 \sin\omega t\,(e^{wt}) \Longrightarrow He^{wt} = A_1 \int \sin\omega t e^{wt} dt$

Put: $\int e^{wt}\sin\omega t dt = W \quad \Longrightarrow Ee^{wt} = A_1 W$

Integrating by parts, $\int u dv = vu - \int v du$: Put $u = e^{Qt} \Longrightarrow du = e^{Qt}$

$dv = \sin\omega t \Longrightarrow v = -\frac{\cos\omega t}{\omega}$

$$W = -\frac{e^{Qt}\cos\omega t}{\omega} + \int \frac{Q}{\omega}\cos\omega t e^{Qt}\, dt$$

$$= -\frac{e^{Qt}\cos\omega t}{\omega} + \frac{Qe^{Qt}}{\omega^2}\sin(\omega t + \varphi) + \frac{Q}{\omega}\int \frac{Q}{\omega}\sin\omega t e^{Qt}\, dt$$

$$W = -\frac{e^{Qt}\cos\omega t}{\omega} + \frac{Qe^{Qt}}{\omega^2}\sin\omega t + \frac{WQ}{\omega^2}$$

$$W\left(1 - \frac{Q^2}{\omega^2}\right) = -\frac{e^{Qt}\cos\omega t}{\omega} + \frac{Qe^{Qt}}{\omega^2}\sin\omega t$$

$$W = -\frac{Qe^{Qt}\cos\omega t}{\omega^2 + Q^2} + \frac{Qe^{Qt}}{\omega^2 + Q^2}\sin\omega t + K$$

$$Ee^{Qt} = \frac{A_1 Qe^{Qt}\cos\omega t}{\omega^2 + Q^2} + \frac{Qe^{Qt}}{\omega^2 + Q^2}\sin\omega t + K$$

$$E(t) = -\frac{A_1 Q\cos\omega t}{\omega^2 + Q^2} + \frac{Q}{\omega^2 + Q^2}\sin\omega t + Ke^{Qt}$$

Initial condition $E(0) = C_2$ $(constant)$, from equation the above

$$-\frac{A_1 Q\cos\omega t}{\omega^2 + Q^2} + \frac{Q}{\omega^2 + Q^2}\sin\omega t + K = C_2$$

$$K = \frac{A_1 Q \cos \omega t}{\omega^2 + Q^2} - \frac{Q}{\omega^2 + Q^2} \sin \omega t + C_2$$

$$E(t) = -\frac{A_1 Q \cos \omega t}{\omega^2+Q^2} + \frac{Q}{\omega^2+Q^2} \sin \omega t + e^{Qt}\left(\frac{A_1 \mathrm{Q} \cos \omega t}{\omega^2+Q^2} - \frac{\mathrm{Q} \sin \omega t}{\omega^2+Q^2} + C_2\right) \quad (12)$$

From all indications, the solutions of equations (1) and (2) given in (11) and (12) showed that there cheap goods and expensive products enters the market (diffusion), progression and spreads for continuous enlargement of any domain or company for individual or national development. Equations (11) and (12) revealed the effect of national economy growth or inflation in the production of both goods showed periodic pattern dynamics.

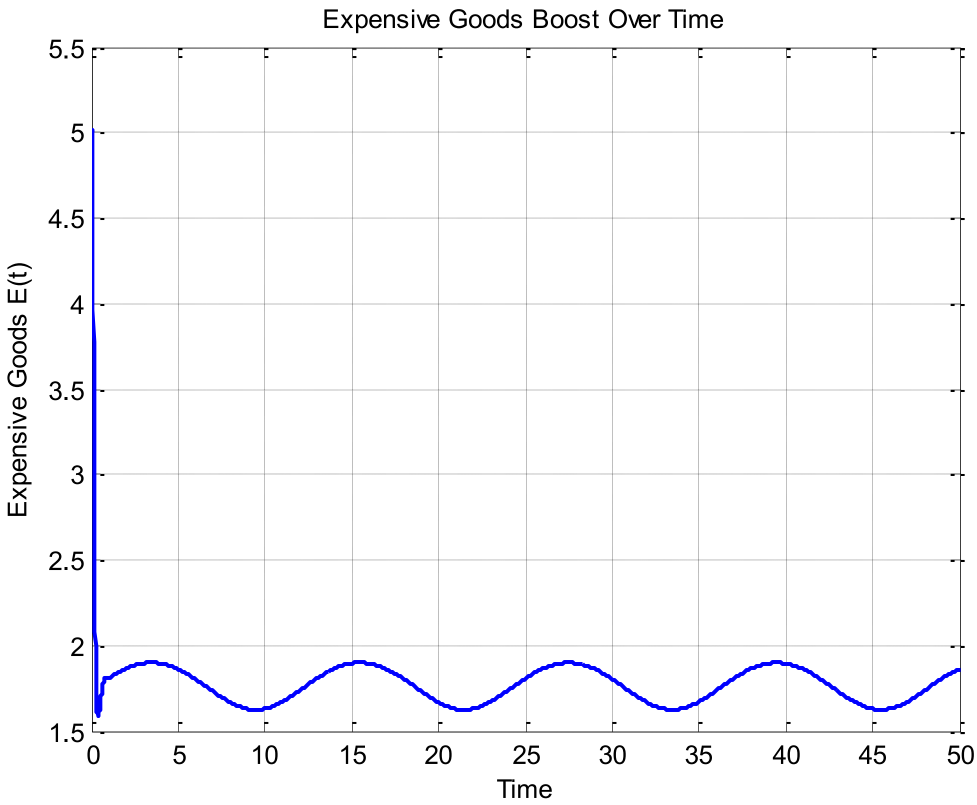
Table 1. Values of the model parameters, description and units

Variables	Description	Value	Units	Source
C	State variable –Cheap Goods		**Dozens, cartons, rolls**	Assumed
E	State variable –Expensive Goods		**Dozens, cartons, rolls**	Assumed
T	Time independent variable		Days	Assumed
$\frac{\partial}{dt}$	Partial derivatives with respect to time			Assumed
r_1	Rate of production of cheap goods	10,12,24, 36,40,…	**Dozens, cartons, rolls**	Assumed
r_2	Rate of production of expensive goods	10,12,24,	**Dozens, cartons, rolls**	
Q	Quantity of expensive goods that enters the market per time	10,12,24, 36,40,…	**Dozens, cartons, rolls**	
D_{12}	cross diffusion for cheap goods and expensive goods in the same market	1, 2, 3,4….	**Dozens, cartons, rolls**	
D_{21}	cross diffusion for expensive goods and cheap goods in the same market	1, 2, 3,4….	**Dozens, cartons, rolls**	
k	Rate of entering into the market dynamics	1, 2, 3,4	**Dozens, cartons, rolls**	
q	Profit maximization for cheap goods	0.7	**Dozens, cartons, rolls**	
h	Competition coefficient for cheap goods	0.8	Dimensionless	
f	Force applied for cheap goods	2		Assumed

∇^2	Second order Laplacian operator in 3-dimension for the cheap and expensive goods			Assumed
ρE	number of cheap goods against expensive goods that enters market per time	1,2, 3,4,		
ρ	measure of the quality of expensive goods	1,2, 3…		
$\delta E/(\beta + E)$	Rate at which cheap goods outweighs expensive goods out of the market- the Holling-type II predator response, δ-maximum demand of expensive goods that can be sold per cheap goods per time, β-saturation value of expensive goods to achieve one half the maximum rate δ	10,12,24, 36,40,…	**Dozens, cartons, rolls**	
ω	Angular frequency	$\frac{2\pi}{12}$	Radians/sec	Assumed
$\emptyset$	Phase shift	$\pm\frac{\pi}{4}$	Radians	Assumed
β	Saturation value of hard skills to achieve one half the maximum rate δ	2,3,4,5….	dimensionless	
μ	Expired expensive goods	1, 2, 3, 4,….	**Dozens, cartons, rolls**	Estimated
A	Amplitude of the noise for Cheap goods	1		
$A\sin(\omega t + \emptyset)$	Noise and periodic force term for Cheap goods		Radians	
$B\sin\omega t$	Noise and periodic force term for Expensive goods			
B	Amplitude of the noise for Expensive goods	1		

$W(x-kt)$	Travelling wave equations in the positive x-direction			
∇	Laplacian operator in 3-dimensions			
$\nabla^2 C$	$9(x_i - kt)$	Use the values of x, k, t		Assumed
$\nabla^2 E$	$9(x_i - kt)$	Use the values of x, k, t		
x	Position of the goods in the market as a function of $x = x_1, x_2, x_3$	1, 2, 3,4,5,6,…	Metres	Assumed

Results
Simulations



The simulation graph shows the dynamic behaviour of expensive goods E(t) over time within the framework of the marketing dynamics model of fair and unfair competition that you formulated. In this model, expensive goods behave like the prey population, while cheap goods behave like the predator, following a structure similar to the Holling–Tanner predator–prey interaction. The behaviour seen in the graph reflects how expensive goods evolve in the market under the influence of production, competition from cheap goods, expiration of goods, and external forcing such as human activities and noise.

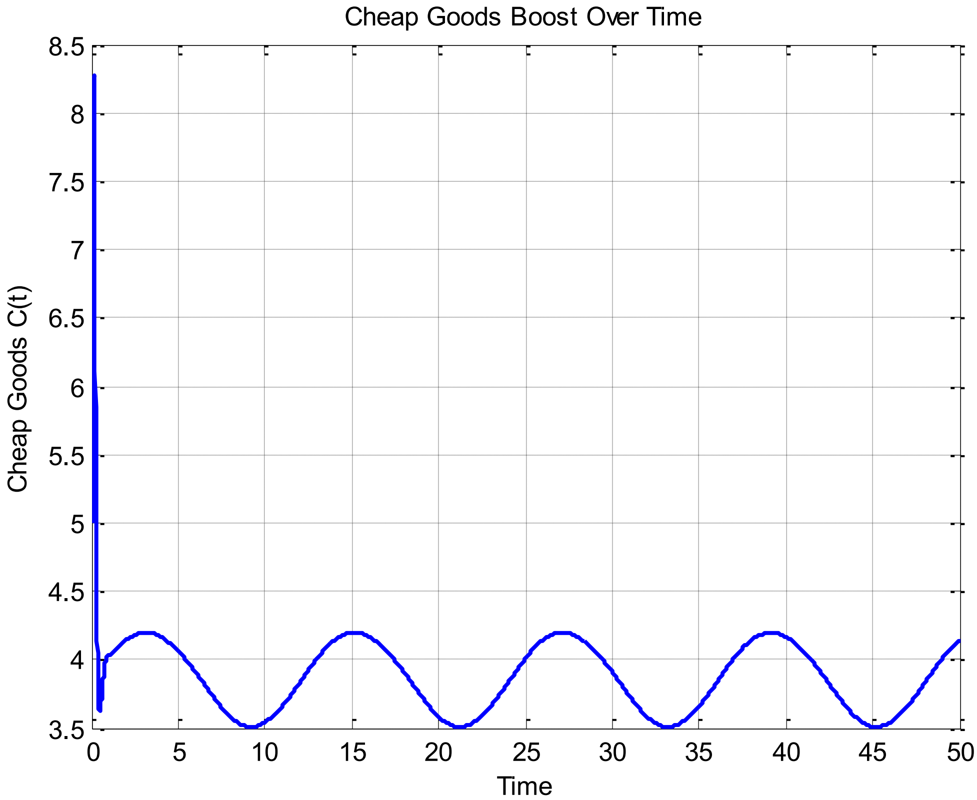
At the beginning of the simulation, the value of expensive goods starts from an initial condition of approximately E(0)=5. Immediately after the simulation begins, the system rapidly adjusts from this initial state. This adjustment occurs because the market dynamics attempt to balance the different forces acting on the expensive goods. The production term $r2E(1-\frac{E}{Q})$ increases the quantity of expensive goods entering the market, but at the same time the competitive interaction term $\frac{\delta CE}{\beta+E}$ represents how cheap goods push expensive goods out of the market. In addition, the decay term $-\mu E$ represents the expiration or loss of expensive goods over time. Because of these competing forces, the system quickly drops from the initial high value toward a lower equilibrium region. After this short adjustment period, the expensive goods stabilize around a value close to 1.7–1.9. However, the curve does not become completely flat. Instead, it shows regular oscillations over time. These oscillations occur because the model includes a periodic forcing and noise term $B\sin(\omega t)$. This sinusoidal component represents fluctuations in the market environment, such as seasonal demand, promotional activities, market disturbances, or other external influences that affect the availability and sales of expensive goods. In the model description you saved, this type of fluctuation was described as red noise, which captures external disturbances affecting the expensive goods subsystem.

The oscillatory behaviour therefore reflects a dynamic market equilibrium rather than a static one. The expensive goods do not settle at a constant level; instead they fluctuate around a stable average value due to the periodic forcing applied to the system. This indicates that even when production and competition forces reach a balance, external market forces such as advertising campaigns, promotional cycles, consumer trends, or environmental disturbances can continuously perturb the system.

Another important observation from the graph is that the oscillations remain bounded and stable throughout the entire time interval from 0 to 50. The amplitude of oscillation remains small, indicating that the system is dynamically stable. In economic interpretation, this suggests that although cheap goods exert competitive pressure on expensive goods through the Holling type II response, the expensive goods are not completely eliminated from the market. Instead, they persist at a lower but stable level. This reflects a realistic market situation where cheaper alternatives dominate a market but premium or expensive goods still maintain a consistent presence due to quality differences, brand perception, or niche demand.

The role of the quality parameter ϱ and the saturation parameter β also contributes to this stability. These parameters regulate how strongly cheap goods can displace expensive goods from the market. Because the displacement term $\frac{\delta CE}{\beta+E}$ saturates as E increases, the competitive pressure does not grow indefinitely. This saturation effect prevents the system from collapsing completely and allows the coexistence of both cheap and expensive goods in the marketplace.

From the perspective of your marketing dynamics model, this simulation demonstrates that the interaction between production, competition, human actions, and environmental noise leads to periodic fluctuations around a stable market state. The expensive goods maintain a persistent but moderated presence in the market despite the pressure exerted by cheap goods. The sinusoidal oscillations further illustrate that real-world markets rarely remain perfectly steady; instead they exhibit continuous cycles influenced by external factors such as promotions, advertising campaigns, and consumer behaviour patterns. the graph confirms that the expensive goods subsystem reaches a stable oscillatory equilibrium under the influence of competition with cheap goods, expiration effects, and periodic noise forcing. This behaviour supports the theoretical formulation of the model, showing how external market disturbances and competitive interactions jointly shape the long-term dynamics of goods circulating in the marketplace.



The simulation graph illustrates the temporal dynamics of cheap goods (t) in the marketing system described in your saved model formulation on marketing dynamics of fair and unfair competition with human action, noise, and diffusion effects. In this framework, cheap goods are modeled as the predator component, while expensive goods represent the prey component, following the Holling–Tanner type predator–prey interaction. The behaviour observed in the graph reflects how cheap goods evolve in the market under the combined influence of production mechanisms, interaction with expensive goods, human marketing activities, and periodic environmental disturbances.

At the beginning of the simulation, the cheap goods start with an initial value close to (0)=5. Immediately after the start of the simulation, the system experiences a rapid adjustment. This occurs because the market is initially out of balance and the governing forces in the model quickly drive the system toward its natural dynamic behaviour. The production term $r1(1-\frac{C}{\rho E})$regulates the growth of cheap goods based on the availability of expensive goods in the market. Since cheap goods benefit from the presence of expensive goods, this interaction term limits the growth of cheap goods as their quantity increases relative to expensive goods. After the short transient adjustment phase, the cheap goods settle into a stable oscillatory pattern. The values fluctuate approximately between 3.5 and 4.2 throughout the simulation period. These oscillations arise primarily due to the periodic forcing and noise term $A\sin(\omega t+\phi)$ included in the governing equation for cheap goods. In the theoretical formulation of your model, this sinusoidal term represents green noise, which captures external fluctuations affecting the cheap goods subsystem. Such fluctuations may represent variations in market promotion strategies, advertisement campaigns, consumer buying patterns, or seasonal marketing influences.

Another factor contributing to the behaviour of cheap goods is the human action term qhfC included in the model. This term reflects the combined effect of profit maximization Since these human-driven factors positively influence the promotion and distribution of cheap goods, they provide a continuous boost that helps maintain the presence of cheap goods in the market. This explains why the cheap goods maintain a slightly higher stable level compared to expensive goods in the earlier simulation. The human action term essentially strengthens the competitive advantage of cheap goods within the market structure.

The oscillatory pattern seen in the graph therefore represents a dynamic equilibrium state rather than a fixed equilibrium. The cheap goods do not converge to a constant value; instead, they fluctuate regularly around an average level. This behaviour reflects realistic economic markets where supply, demand, marketing activities, and external disturbances continually interact. Even when the system reaches a stable state, periodic external influences prevent it from remaining perfectly constant.

The bounded nature of the oscillations also indicates that the system is stable under the given parameter values. The cheap goods neither grow without bound nor collapse to zero. Instead, they persist at a sustainable level in the market. This result supports the conceptual interpretation of the model that cheap goods dominate the competitive interaction due to strong marketing forces and human actions, but their growth is still regulated by the interaction with expensive goods and by the logistic-type production structure. the graph demonstrates that cheap goods maintain a stable and persistent presence in the market, supported by human marketing actions and periodic environmental influences. The oscillations observed over time illustrate how external noise and market forces continuously shape the dynamics of goods distribution. Within the context of the proposed marketing dynamics model, this behaviour confirms that cheap goods can sustain their dominance in the market while still experiencing cyclical fluctuations driven by external disturbances and competitive interactions with expensive good.

Table 2 Effect of Reducing Noise on Cheap and Expensive Goods

Noise A	Cheap Goods C	Expensive Goods E
9	3.9808	NaN
8	3.9980	NaN
7	4.0158	NaN

6	4.0342	NaN
5	4.0534	NaN
4	4.0732	NaN
3	4.0938	2.1068
2	4.1151	1.9887
1	4.1372	1.8622

The table presents the effect of reducing the noise amplitude on the dynamics of cheap goods C and expensive goods E within the marketing dynamics model of fair and unfair competition that you formulated. In the model equations, the noise term is introduced through the sinusoidal forcing components $A\sin(\omega t+\phi)$ for cheap goods and $B\sin(\omega t)$ for expensive goods. These sinusoidal components represent environmental disturbances, fluctuations in consumer demand, irregular promotional activities, and other external market forces that periodically influence the availability and sales of goods in the marketplace.

From the results shown in the table, the noise amplitude is gradually reduced from 9 down to 1. As the noise intensity decreases, the response of cheap goods and expensive goods in the market becomes clearer. For the cheap goods C, the values increase steadily from approximately 3.9808 to 4.1372 as the noise amplitude decreases. This indicates that when the market experiences strong external disturbances (high noise levels), the quantity of cheap goods in circulation is slightly suppressed. As the disturbances reduce, the cheap goods stabilize at higher levels. This behaviour reflects the role of noise as a destabilizing factor in the market environment. High fluctuations disrupt normal supply and distribution processes, causing slight reductions in the effective presence of cheap goods in the market.

For the expensive goods E, the behaviour is somewhat different. When the noise amplitude is high (from 9 down to 4), the simulation produces NaN values, which indicates that the numerical solution becomes unstable or undefined under very strong disturbance conditions. In practical terms, this suggests that extremely high noise levels can destabilize the expensive goods subsystem in the market. Because expensive goods already face competitive pressure from cheap goods through the Holling-type II interaction term $\frac{\delta CE}{\beta+E}$, the addition of strong external disturbances can push the system beyond a stable operating range, leading to numerical divergence in the simulation.

However, when the noise amplitude is reduced to 3, 2, and 1, the system begins to produce stable values for expensive goods. At these lower disturbance levels, the expensive goods settle around values of approximately 2.1068, 1.9887, and 1.8622 respectively. This indicates that when market fluctuations become moderate, the expensive goods are able to maintain a stable presence in the market, although their levels remain lower than those of cheap goods due to competitive disadvantage.

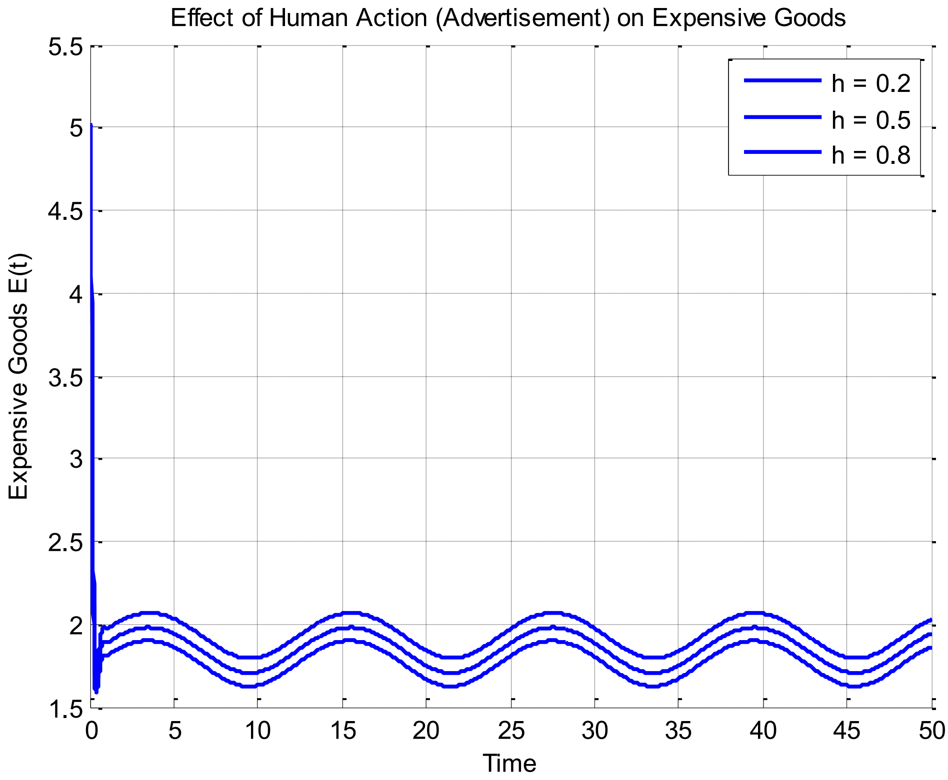
The results therefore reveal an important economic interpretation. High market noise strongly destabilizes expensive goods, while cheap goods remain relatively resilient due to the additional support from the human action term qhfC, which represents marketing forces such as advertisement, profit incentives, and selling strategies. Cheap goods therefore maintain a stable presence even under high disturbance levels, whereas expensive goods are more sensitive to environmental fluctuations.

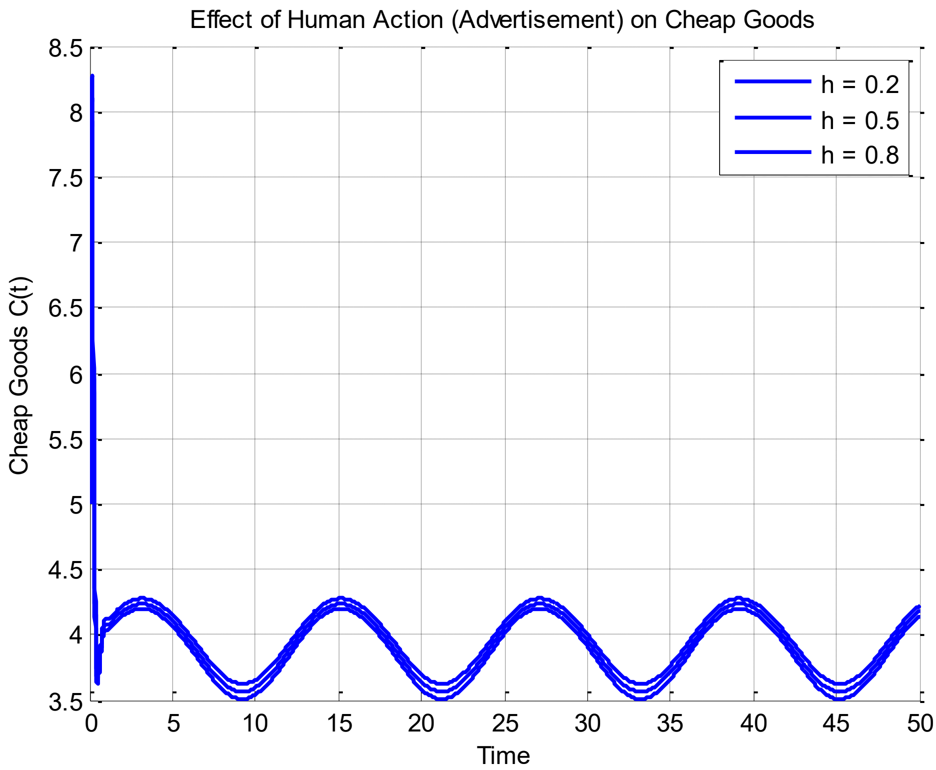
Another observation from the table is that as the noise amplitude continues to decrease toward lower values, both goods approach more stable equilibrium levels. Cheap goods gradually increase toward approximately 4.14, while expensive goods stabilize below 2. This reflects the underlying structure of the competitive model where cheap goods dominate the market due to stronger marketing forces and higher accessibility, while expensive goods survive at a lower but persistent level.

the simulation results demonstrate that reducing noise in the market environment improves the stability of both cheap and expensive goods, although the improvement is more pronounced for expensive goods. High levels of disturbance lead to instability and unpredictable behaviour in the system, whereas lower noise levels allow the market dynamics to settle into a stable oscillatory equilibrium. These findings highlight the importance of controlling external disturbances and market fluctuations in maintaining a balanced and sustainable competitive marketplace.

Table 3. Effect of Human Action on Goods

Human Action h	Cheap Goods	Expensive Goods
0.20	4.2172	2.0264
0.40	4.1891	1.9686
0.60	4.1625	1.9140
0.80	4.1372	1.8622
1.00	4.1131	1.8132


Effect of Human Action (Advertisement) on Expensive Goods
Expensive Goods E(t)
Time
h = 0.2
h = 0.5
h = 0.8
5.5
5
4.5
4
3.5
3
2.5
2
1.5
0
5
10
15
20
25
30
35
40
45
50


Effect of Human Action (Advertisement) on Cheap Goods
Cheap Goods C(t)
Time
h = 0.2
h = 0.5
h = 0.8
8.5
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
0
5
10
15
20
25
30
35
40
45
50

Conclusion

In this study, cheap goods are more influenced by advertisement, sellers and buyers combined together is (human action and diffusion). Here the demand for cheap goods is often more directly affected by consumer behavior, trends, and preferences. Consumers are more likely to respond quickly to price changes, promotions, or marketing strategies. Human decisions on purchasing habits, influenced by factors such as income, advertising, and social dynamics, can significantly impact the market for cheap goods. However, cheap goods often spread rapidly through social networks and word-of-mouth since they require lower financial commitment. The adoption of inexpensive items can occur faster due to lower barriers to entry for consumers, which is modelled using diffusion processes. Expensive Products are also influenced by human actions (given that branding, prestige, and luxury appeal are vital), their diffusion is typically slower due to higher costs, more significant decision-making processes, and specific target markets. Since both cheap and expensive products are influenced by human actions, cheap goods show a stronger correlation with rapid changes driven by those actions and diffusion dynamics.

This study revealed that the marketing dynamics model reveals critical insights into consumer behavior and market strategies for both cheap and expensive products. For cheap goods, the model highlights the importance of price sensitivity, volume sales, and accessibility, often driven by features such as convenience and word-of-mouth. On the other hand, expensive products rely more on perceived value, brand loyalty, and emotional connection, emphasizing quality and exclusivity in their marketing approaches. Understanding these dynamics allows businesses to tailor their strategies effectively, optimizing pricing, promotion, and distribution to meet the distinct needs of each market segment. By leveraging insights from both ends of the pricing spectrum, companies can enhance their competitive advantage, drive sales, and ultimately achieve sustainable growth in an increasingly diverse marketplace.

The marketing dynamics model reveals that the interplay between cheap goods and expensive products significantly influences consumer behavior and market trends. While cheap goods attract price-sensitive customers, expensive products often symbolize luxury, quality, and status. To maximize market share, businesses should adopt a dual strategy: offering affordable options to capture the mass market and premium products to target high-end consumers. By understanding these dynamics, companies can effectively position their brands, tailor marketing efforts, and ultimately drive growth in a competitive landscape.

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